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Noise estimation and power spectrum analysis using different window techniques

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Abstract: *Achieving quality resolution in power spectrum estimation (PSE) can be accomplished by deciding on the optimal size of the data sample. This paper has been putforth to present the PSE, being performed on a data length with diverse windows. This estimation has been executed, by putting a nonparametric method into action i.e., Welch method. The periodogram technique supported on Welch method is capable of rendering good resolution if the optimal selection of data length samples is done. The PSE of Rectangular, Triangular, Hamming, Hanning, Blackmann, Kaiser and Chebyshev window has been designed and simulated using MATLAB.*

Index Terms: *FIR systems, power spectrum, windows*

I. Introduction

The signals encountered in majority of the applications are such that the variability of that particular signal at that very instant can be known, with its deviation in the future can only be predicted with a very little accuracy. Only probabilistic statements are feasible to make about that variation. In order to get a picture of this, a mathematical device is best suited to describe such a signal, an ensemble of possible realizations of a random sequence, having some associated probability of occurrence of the signal at each instant. Though, the entire ensemble of realizations can only provide the observer a single realization of the signal; it then might be assumed that the probabilistic assumptions of the signal at its previous instant can be taken unchanged to the present instant. However, it cannot be assumed because the realizations of a random signal, seen as discrete time sequences, doesn't possess finite energy, and therefore cannot have DTFTs. A random signal generally has measurable average power and, hence, can be characterized by an average power spectral density. For our convenience, in short, we will use the name power spectral density (PSD) for that quantity.

In order to estimate the PSD of a signal, we use different techniques that which can be classified broadly into parametric and non-parametric methods. To analyze a random signal, non-parametric methods are the most suitable, as it utilizes the periodogram function in the analysis [1]. The modification in this analysis is the reduction of other unwanted noise and extra harmonics, it is done using a window function. Window functions most primarily smoothens the signal and presents the signal to its limited length [2].

II. Analysis Of A Signal Using Window Function

Signals that we mostly come across in our day to day life are digital signals, which are infinite or adequately vast that the manipulation of the dataset as a whole is presumably a complex issue. Statistical analysis also is laborious for a finitely large signal, because, it requires every point to have a complete view of the signal. So as to avoid these kind of problems, a process called **windowing** is developed [3], in which many small subsets of data is used to represent the whole signal. Windowing is a process in which a larger data set is split into many smaller subsets so as to process and analyze the signal. Applying the windowing process to a dataset alters its spectral properties.

Now, let us observe how this process happens in different kinds of windows. A general form of inducing a system with a window is given below [4]:

Suppose a system *H(z)*, with input *X(z)* and output *Y(z)*; can be modeled as:

$$Y(z) = X(z)W(z) --- (1)$$

If the transfer function of a window is given by *W(z)*, then the mathematical application of the window to our signal, *X(z)* will be:

$$\widehat{X}(z) = X(z)W(z) --- (2)$$

Then the windowed signal can be induced into our system, *H(z)* as like [5]:

$$\widehat{Y}(z) = \widehat{X}(z)H(z) --- (3)$$

Rectangular window

A rectangular window also known as the **boxcar** or **drichlet** window, is the simplest window [6], in which, all the data points are truncated that which are present outside the window and are hence assumed to be zero. High-frequency components will get introduced into the system at the cut-off points of the sample, it is equivalent to replacing all the values outside the part of the signal that which the window does not cover to zero and the values of a data sequence to its original values inside the window, making it appear as though the waveform suddenly turns on and off. Let the window function represented as:

$$w(n) = 1 --- (4)$$

Triangular window

Triangular window is popularly also known as **Bartlett** or **fejer** window. It is generally given by:

$$w(n) = 1 - \left| \frac{n - \frac{N-1}{2}}{\frac{L}{2}} \right| --- (5)$$

where L can be N, N+1 or N-1. All three values of L converge at large N.

It can be seen as a convolution of two rectangular windows with a width of N/2. The Fourier transform of the result is the squared values of the transform of the half-width rectangular window.

Hann window

Hanning window can also be called '**raised cosine window'.** It is defined as:

$$w(n) = 0.5\left(1 - \cos\left(\frac{2\Pi n}{N-1}\right)\right) = hav\left(\frac{2\Pi n}{N-1}\right) --- (6)$$

In this window the ends of the cosine function just touch zero, so at about 18 dB per octave the side-lobes roll off.

Hamming window

The hamming window is designed so as to optimize the maximum (nearest) side lobe to its minimum value, minimizing it to a height of about one-fifth of the Hann window [7]. It is given by:

$$w(n) = \alpha - \beta \cos\left(\frac{2\Pi n}{N-1}\right) --- (7)$$

with

$$\alpha = 0.54, \beta = 1 - \alpha = 0.46 --- (8)$$

Blackmann window

Blackman window smoothen the signal at its third and fourth side-lobes [8]. These windows are defined as:

$$w(n) = a_0 - a_1 \cos\left(\frac{2\Pi n}{N-1}\right) + a_2 \cos\left(\frac{4\Pi n}{N-1}\right) --- (9)$$

$$a_0 = \frac{1-\alpha}{2}; a_1 = \frac{1}{2}; a_2 = \frac{\alpha}{2} --- (10)$$

Kaiser window

The kaiser window also called Kaiser-Bessel window is a simple approximation of the DPSS window using Bessel functions [9], this window is discovered by Jim Kaiser and are given as:

$$w(n) = \frac{I_0\left(\Pi\alpha\sqrt{1 - \left(\frac{2n}{N-1} - 1\right)^2}\right)}{I_0(\Pi\alpha)} --- (11)$$

Chebyshev window

The chebyshev window also known as dolph-chebyshev window, for a given main lobe width minimizes the chebyshev norm of the side-lobes.

The zero-phase Dolph–Chebyshev window function $w_0(n)$ is usually defined in terms of its real-valued discrete Fourier transform [8], $W_0(k)$:

$$W_0(k) = \frac{\cos\left\{N\cos^{-1}\left[\beta\cos\left(\frac{\Pi k}{N}\right)\right]\right\}}{\cosh\left[N\cosh^{-1}(\beta)\right]} --- (12)$$

$$\beta = \cosh\left[\frac{1}{N}\cosh^{-1}\left(10^{\alpha}\right)\right] --- (13)$$

where the parameter α sets the Chebyshev norm of the side-lobes to -20α decibels.

The window function can be calculated from $W_0(k)$ by an inverse discrete fourier transform (DFT):

$$w_0(n) = \frac{1}{N}\sum_{k=0}^{N-1} W_0(k).e^{i2\Pi kn/N}, -N/2 \le n \le N/2 --- (14)$$

The *lagged* version of the window, with $0 \le n \le N-1$, can be obtained by:

$$w(n) = w_0\left(n - \frac{N-1}{2}\right) --- (15)$$

Estimation of noise in a power spectrum density

For the estimation of noise of a random signal from its power spectrum density, we first need to analyze the signal. In real-time applications, the signal is mostly a random kind of signal i.e., most of its parameters are unknown to analyze; in this case, we go for the non-parametric analysis of the random signal. In the non-parametric analysis of the signal, welch method is the most popular method of analysis [10].

Welch method is widely preferred for the analysis of a random signal as it gives the most appropriate and precise analysis of the signal after the analysis being carried out. This method estimates the power spectrum of the signal at different frequencies, by simply converting the time domain of the signal into its respective frequency domain. It mostly got designed from the Bartlett method which had its improvements over the standard periodogram method of spectrum estimation in terms of noise reduction as it has a better edge in cutting down the noise of the random signal. The reduction in noise is done in compensation to the resolution of the frequency.

In this method, the entire signal is split into data segments; the number of data segments to be split depends on the length of the signal. The split overlapping segments are windowed individually; the influence of the window is more on the data that is present at center of the segment. The data at the edges of the segment does not get influenced by the window and hence suffer a loss of data which can be mitigated by the other overlapping segment of the data set. The entire windowing process is carried out in time domain.

Discrete fourier transform is applied to the above obtained magnitude of the signal, so as to get the periodogram and it is squared. The individual squared periodograms for every split segment are calculated and then averaged; this reduces the variance of a random signal and increases the accuracy of the analysis. From the obtained power spectrum of the random signal, the maximum distortion of the signal can be observed and can be said to be the noise of that signal

III. Results

The obtained results for different windows are carried out on the following data:

Sampling frequency, f_s= 500 Hz; length of the data sequence, n= 500; length of FFT= 250.

Rectangular window:

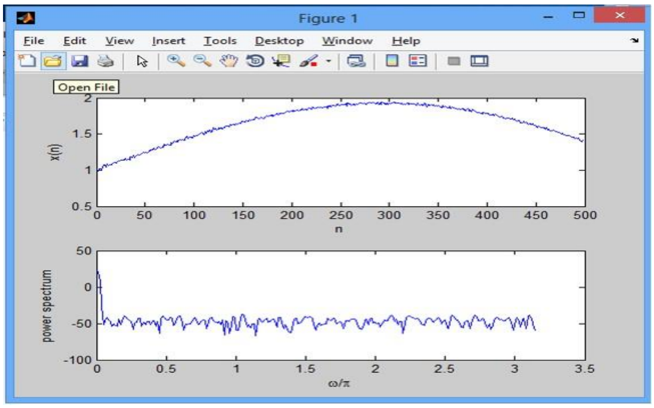

Fig. 1. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of rectangular window (bottom)
The peak of the signal will be obtained at 291st sample with the magnitude of the signal being 1.9517 (top fig.).
Peak value of the noise is -69.2 units; and is found at 1.960 units (bottom fig.).

Triangular window:

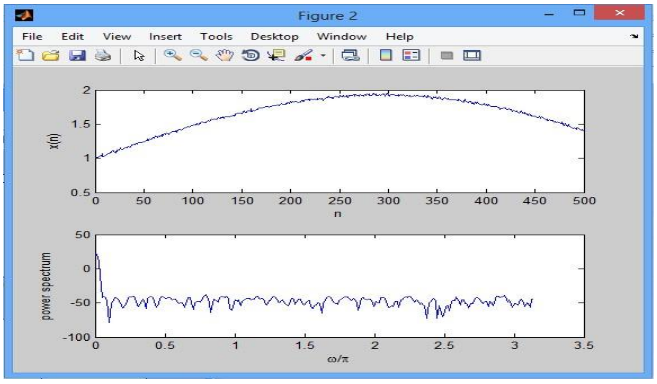

Fig. 2. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of triangulaer window (bottom)
The peak of the signal will be obtained at 286th sample with the magnitude of the signal being 1.96 (top fig.).
Peak value of the noise is -68 units; and is found at 1.533 units (bottom fig.).

Hanning window:

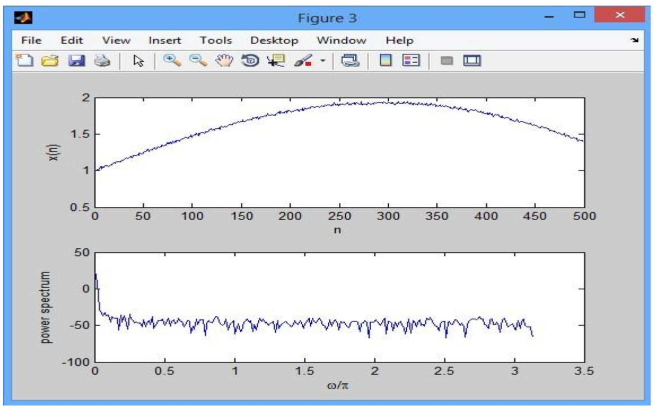

Fig. 3. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of hanning window (bottom)
The peak of the signal will be obtained at 309th sample with the magnitude of the signal being 1.9447 (top fig.).
Peak value of the noise is -64.5 units; and is found at 2.484 units (bottom fig.).

Hamming window:

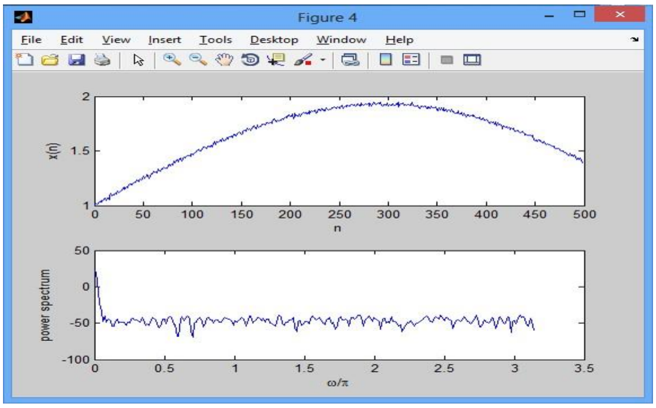

Fig. 4. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of hamming window (bottom)
The peak of the signal will be obtained at 278th sample with the magnitude of the signal being 1.94 (top fig.).
Peak value of the noise is -67.3 units; and is found at 2.513 units (bottom fig.).

Kaiser window:

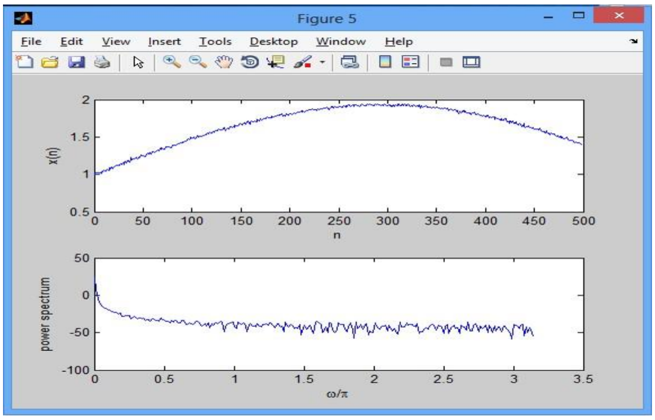

Fig. 5. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of kaiser window (bottom)
The peak of the signal will be obtained at 313rd sample with the magnitude of the signal being 1.9542 (top fig.).
Peak value of the noise is -70.5 units; and is found at 2.35 units (bottom fig.).

Blackmann window:

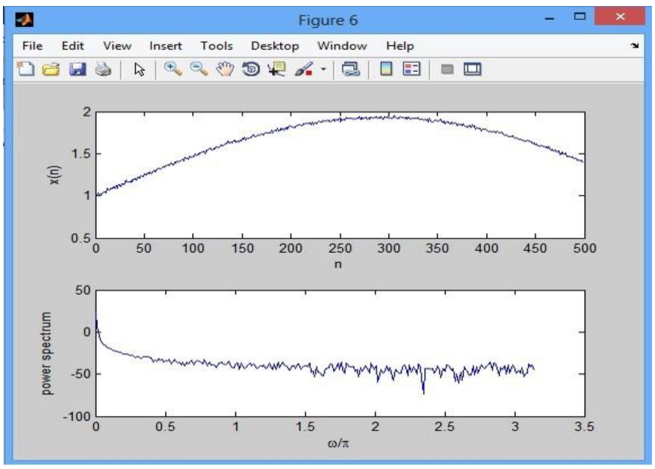

Fig. 6. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of blackmann window (bottom)
The peak of the signal will be obtained at 315th sample with the magnitude of the signal being 1.9455 (top fig.).
Peak value of the noise is -76.1 units; and is found at 2.45 units (bottom fig.).

Chebyshev window:

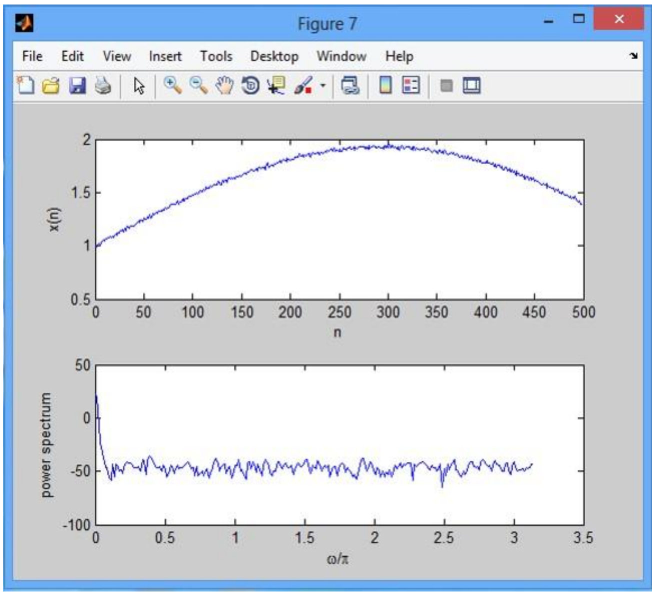

Fig. 7. Variation of the signal with the length of the sequence (top);

Response of the signal with the application of chebyshev window (bottom)
The peak of the signal will be obtained at 283rd sample with the magnitude of the signal being 1.9569 (top fig.).
Peak value of the noise is -90.3 units; and is found at 2.124 units (bottom fig.).

IV. Conclusion

The work presented in this paper, clearly depicts the behavior of the random signal when various windows are applied to it. It can be articulated that, noise value of hanning window being -64.5 units of power spectrum, has the minimum noise levels in comparison to other windows; chebyshev window having a maximum magnitude of 1.9569 the signal obtained at its 283rd sample of the entire signal samples. It can also be concluded that for such a kind of random signal, in applications where low noise is necessary, hanning window can be used and chebyshev window can be applied to similar kind of signal where maximum magnitude of the signal is required at lower number of data samples.

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